Weak Matrix Elements: Hunting New Physics with the Lattice

Amarjit Soni
High Energy Theory, Brookhaven National Laboratory
Upton, NY 19973

Outline

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2) Exact Chiral Symmetry on the Lattice

3) $K \to \pi\pi$ & ε'/ε: Progress and Outlook

4) Heavy-Light Matrix Elements

5) B versus K Unitarity Triangle

6) Other Important Applications: Lowest Order Hadronic Contribution to $(g-2)_\mu$

7) Summary

Most of the lattice computing effort is under the auspicious of the RBC (RIKEN-BROOKHAVEN-COLUMBIA) Collaboration.

Why Lattice is Needed

Due to the non-perturbative nature of low energy QCD, many experimental results, often attained at enormous cost cannot be used effectively to test the Standard Model unless accurate values of hadronic matrix elements are known; lattice is the only reliable tool for such calculations

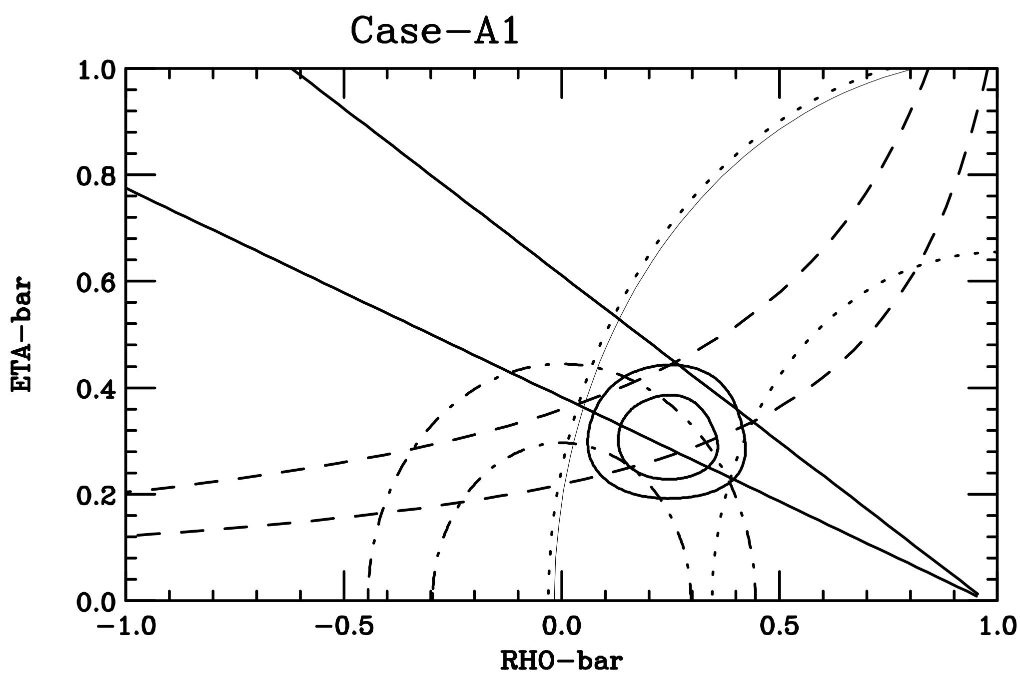
$|\varepsilon_K|$ (BNL '64; Christenson et al), provides a CLASSIC EXAMPLE.

$$|\varepsilon_K| = \hat{B}_K C_K \lambda^6 A^2 \bar{\eta} \{\eta_1 S(x_c) + \eta_2 S(x_t)[A^2 \lambda^4 (1-\bar{\rho})] + \eta_3 S(x_c, x_t)\} \qquad C_K = \frac{G_F^2 f_K^2 m_K m_W^2}{6\sqrt{2}\pi^2 \Delta m_K}$$

The experimentally known value $|\varepsilon_K| = 2.27 \times 10^{-3}$ can be used to extract information on the poorly known SM parameters $\bar{\rho}$ and $\bar{\eta}$, once the non-perurbative quantity, B_K becomes known, as everything else on the RHS is known quite well.

Besides ε'/ε of special relevance to lattice program here are:

- AGS expt. on $(g-2)_\mu$
- AGS rare K expt on $(K \to \pi\nu\nu)$
- B Factories for anticipated BR$(B \to \rho\nu)$
- Tevatron expts. anticipated measurment of $B_s - \bar{B}_s$ osc. frequency.



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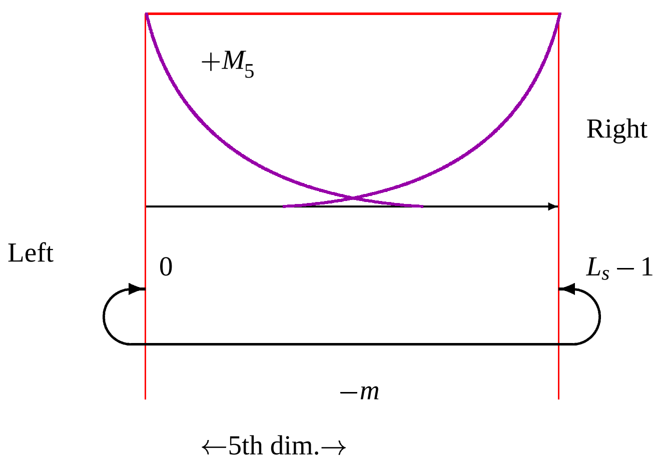
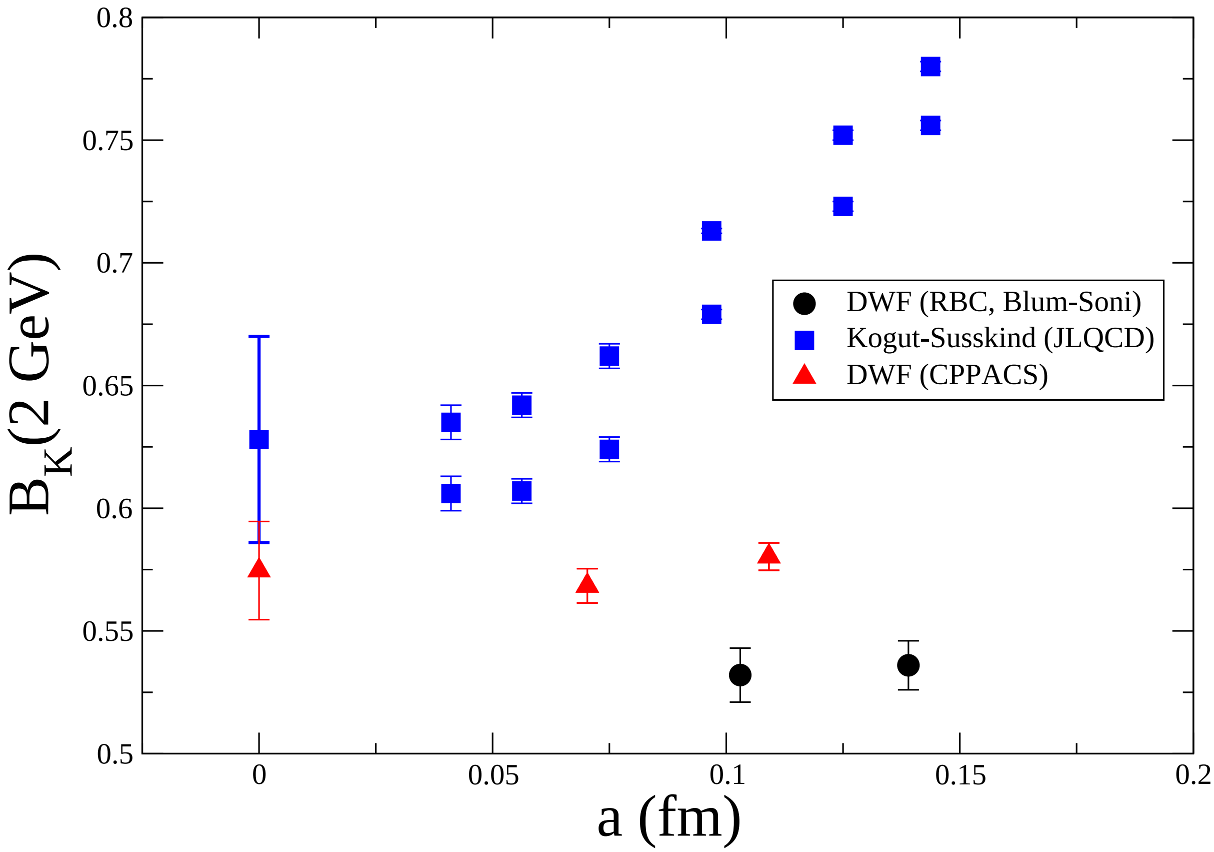
EXACT CHIRAL SYMMETRY ON THE LATTICE

Conventional fermions do not preserve chiral symmetry on the lattice (Nielsen - Ninomiya Theorem)
$\Rightarrow \Delta S = 1, \Delta I = 1/2$ case mixing with lower dim. (power-divergent) operators renders it intractable on lattice.
$\Rightarrow$ mixing of 4-quark operators with wrong chirality ones makes lattice study of $K - \pi$ physics virtually impossible.

Domain Wall Fermions (Kaplan, Shamir, Narayanan and Neuberger)



Practical viabilty of DWF for lattice QCD demonstrated (96-97) in collaboration with Tom Blum
Chiral symmetry on the lattice, $a \neq 0$! Huge improvement
$\Rightarrow$ widespread use at BNL and elsewhere
DWF are "continuum-like" so mixing and renormalization a lot simpler, also significantly improved scaling behavior tends to offset the cost of the extra-dimension.



B_K with DWF's confronts staggered fermion results

Indications are that DWQ answer is 10-15% below the old (staggered) result $\Rightarrow$ tends to correspondingly increase the CP violating phase $\bar{\eta}$ of the SM.

Two Key Steps

I. The $\Delta S = 1$ Effective Hamiltonian in terms of effective local **four-quark operators** $Q_i(\mu)$ with generic **Wilson coefficients** $c_i(\mu)$ including CKM factors.

$$\mathscr{H}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1}^{10} c_i(\mu) Q_i(\mu) \right\}$$

II.Chiral Perturbation theory (χPT) BDSPW'85

Significant technical difficulties associated with $|\pi\pi\rangle$, so use **LO** χPT to relate physical $K \to \pi\pi$ amplitudes to unphysical $K \to \pi$ and $K \to 0$; for example:

$$\langle \pi^+ \pi^- | \Theta_i^{(8,1)} | K^0 \rangle = \frac{4i}{f^3} \left(m_{K^0}^2 - m_{\pi^+}^2 \right) \alpha_i^{(8,1)}$$

Here α's are **LOW ENERGY CONSTANTS** calculable on the lattice via $< \pi|\Theta|K >$, which through above eqns. give all $K \to \pi\pi$ amplitudes to leading order (LO) in χPT

This is a significant approximation that was used in calculations thus far finished, but we know now [**Jack Laiho (PhD student)** and Soni '02] that it can be systematically improved to NLO in future lattice calculations.

Final values for low energy constants

i	$\alpha_{i,\mathrm{lat}}^{(1/2)}$	$\alpha_{i,\mathrm{lat}}^{(3/2)}$
1	$-1.19(31) \times 10^{-5}$	$-1.38(6) \times 10^{-6}$
2	$2.22(16) \times 10^{-5}$	$-1.38(6) \times 10^{-6}$
3	$0.15(113) \times 10^{-5}$	0.0
4	$3.55(96) \times 10^{-5}$	0.0
5	$-2.97(100) \times 10^{-5}$	0.0
6	$-8.12(98) \times 10^{-5}$	0.0
7	$-3.22(16) \times 10^{-6}$	$-1.61(8) \times 10^{-6}$
8	$-9.92(54) \times 10^{-6}$	$-4.96(27) \times 10^{-6}$
9	$-1.85(16) \times 10^{-5}$	$-2.07(9) \times 10^{-6}$
10	$1.55(31) \times 10^{-5}$	$-2.07(9) \times 10^{-6}$

Statistical errors only, for now; work in progress to quantify systematic error.

Note $< \pi|Q_2^{1/2}|K > / < \pi|Q_2^{3/2}|K > \approx 15$ and not O(1)!
$\Rightarrow$ large fraction of the observed enhancement

[Recall $Q_2 \equiv \bar{s}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma_\mu(1-\gamma_5)d$]

FINAL RESULTS

QUANTITY	Experiment	This calculation (statistical errors only)
Re A_0(GeV)	3.33×10^{-7}	$2.96 \pm 0.17 \times 10^{-7}$
Re A_2(GeV)	1.50×10^{-8}	$1.17 \pm 0.05 \times 10^{-8}$
Re A_0/Re A_2	22.2	25.3 ± 1.8
Re $(\varepsilon'/\varepsilon)$	$17 \pm 2 \times 10^{-4}$ (WA)	$-4.0 \pm 2.3 \times 10^{-4}$

($\mu = 2.13$ GeV, One-loop known chiral logs)

- Results show clear $\Delta I = 1/2$ enhancement!

  $\Rightarrow$ (Although further scrutiny and confirmation is desirable and will be coming shortly) it certainly seems that we are very very close to putting a complete end to some 4-decades of speculation on the origin of this enhancement.

- Much more work is needed to improve our calculation of ε'/ε.

WHEN THE DUST SETTLES

$$\left[\frac{\varepsilon'}{\varepsilon} = \left.\frac{\varepsilon'}{\varepsilon}\right|_{I=0} + \left.\frac{\varepsilon'}{\varepsilon}\right|_{I=2}\right] \qquad ; \qquad \text{units } 10^{-4}$$

EXPT $\sim (17 \pm 2)$

OPERATOR	$I = 0$	$I = 2$
Q_4	-4.8 ± 1.1	
Q_6	$\boxed{14.2 \pm 1.9}$	
Q_8	$1.48 \pm .12$	$\boxed{-16.97 \pm .84}$
Q_9		$1.56 \pm .00$

$\Rightarrow$ Although Q_6 and Q_8 are the dominant players, due to the significant cancellations:

- other ops. (e.g. Q_4 & Q_9) are also important and cannot be ignored.
- Accuracy need to be improved before impact on SM can be reliably extracted.
- The experimental value of $\frac{\varepsilon'}{\varepsilon}$ is comparable to $\left.\frac{\varepsilon'}{\varepsilon}\right|_{I=0}$ and $\left.\frac{\varepsilon'}{\varepsilon}\right|_{I=2}$ and not much smaller

$\Rightarrow$ Cancellation NOT between “large numbers”

$\Rightarrow$ Prospects for reliable test of the SM with improvement in accuraccy appear promising.

Regarding the $\Delta I = 1/2$ Rule

	$\mathrm{Re}A_0$ (GeV)	$\mathrm{Re}A_2$ (GeV)
Q_1	$(3.48 \pm .77) \times 10^{-8}$	$(-.363 \pm .016) \times 10^{-8}$
Q_2	$(24.5 \pm 1.6) \times 10^{-8}$	$(1.520 \pm .068) \times 10^{-8}$
Q_6	$(0.050 \pm .006) \times 10^{-8}$	

$\Rightarrow Q_1/Q_2 \sim .14$

$Q_6/Q_2 \leq .01$

$\Rightarrow$ For the $\Delta I = 1/2$ rule Q_2 **[the aboriginal 4-Fermi operator]** is the most important; in particular, Q_6 is completely negligible

$\Rightarrow$ CLEARLY Rules out *SVZ* conjecture

$\Rightarrow$ Repercussions for phenomenological model calculations of ε'/ε. For instance, Bertolini et al "chiral quark model" use $Q_6/Q_2 \approx 20-30\%$.

Improvements for ε'/ε now underway

Previous calculation (hep-lat/010075) had following serious drawbacks:

- I) $a^{-1} \approx 2$ GeV, not fine-grained enough to have a propagating charm quark so it was integrated out.
- II) Quenched approximation was used.
- III) LOχPT was also used.

Now in progress

For I) $\Rightarrow a^{-1} \approx 3$ GeV lattice (quenched) well underway which should allow inclusion of charm as an active flavor

For II) $\Rightarrow a^{-1} \approx 1.7$ GeV Dynamical DW (with 2 sea quarks) lattice simulations also well underway which should give a 1st ever glimpse of quenching effects on weak matrix elements.

For III) $\Rightarrow$ Laiho & AS (hep-ph/0203106) have shown, contrary to previous held belief, that NLOχPT computations of $K \to \pi\pi$ can be done on the lattice. **Indeed prototype calculation will be a part of Jack Laiho's PhD thesis**

Heavy-light Matrix Elements

Excellent chiral behavior of DWQ's makes them very suitable for treatment of light quarks in heavy-light (e.g. B) mesons; in particular, for precision study of SU(3) breaking.

To facilitate precise extraction of $V_{td}/V_{ts} \equiv \lambda^2[(1-\bar{\rho})^2+\bar{\eta}^2]$ two hadronic matrix elements will be targetted, both of which will be needed in conjunction with experimental information expected to become available in the near future.

- $< \rho|\bar{b}\sigma_{\mu\nu}d|B > / < K^*|\bar{b}\sigma_{\mu\nu}s|B >$ which controls the ratio of Br's, $(B \to \rho+\gamma)/(B \to K^*+\gamma)$ and should soon be measureable by the B factories.

- $< B_d|[\bar{b}\gamma_\mu(1-\gamma_5)d]^2|\bar{B}_d > / < B_s|[\bar{b}\gamma_\mu(1-\gamma_5)s]^2|\bar{B}_s >$ which monitors the ratio of oscillation frequencies of $B_s - \bar{B}_s$ and $B_d - \bar{B}_D$, and which the Tevatron experiments should be able to nail down soon.

B versus K Unitarity Triangle

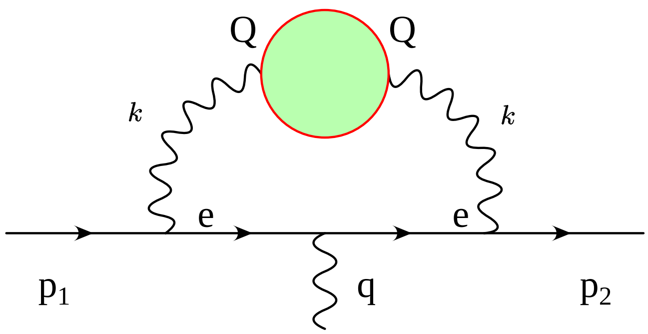
Traditionally, experimental info plus lattice matrix elements are used for ε_K, $B_d - \overline{B}_d$ mass difference (Δm_d) and semi-leptonic $b \to ue\nu$ to constrain ρ and η i.e. the unitarity triangle (UT)

$\Rightarrow$ There ε_K provided crucial (and the only known) CP violation info. However, now that B-factories have seen large CP violation in $B \to \psi K_s$ and it is very "clean", i.e. no hadronic uncertainties, one can replace input from ε_K in the above and construct UT purely from B-physics.

$\Rightarrow$ In the future one can hope to construct another UT purely from K-physics using (greatly improved calculations) of hadronic matrix elements from the lattice, for ε_K, ε' along with improved experimental measurments of $K^+ \to \pi^+ + \nu + \overline{\nu}$ and possibly also $K_L \to \pi^0 + \nu + \overline{\nu}$.

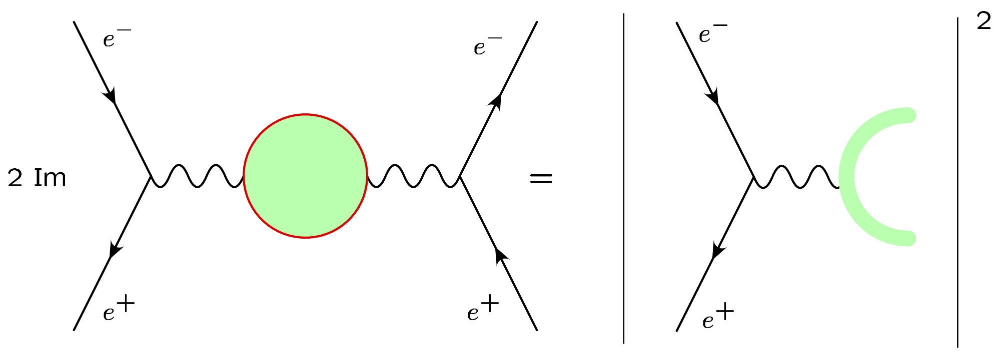
$\Rightarrow$ Comparison of the solutions of the two UT's is likely to become a powerful new avenue to search for new physics.

Lattice calculation of $\mathcal{O}(\alpha^2)$ hadronic vacuum polarization: $\Pi(k^2)$



But the blob, which represents all possible
intermediate hadronic states, is not calculable in perturbation theory (QCD!)

From the optical theorem and the analytic structure of $\Pi(k^2)$, we can compute it from experimental data for $e^+e^- \to$ hadrons cross-section (and $\tau \to \nu_\tau + hadrons$); this is the conventional method.



Tom Blum has recently made a pioneering attempt to calculate this blob directly on the lattice using domain wall quarks.

Results from Tom's pilot study

a^{-1}	V	$a_\mu^{\text{had}}(\alpha^2)$	
1.3	$(2.4\ \text{fm})^3$	460(78) $\text{x}10^{-10}$	
1.3	$(1.2\ \text{fm})^3$	318(69) $\text{x}10^{-10}$	
2.0	$(1.6\ \text{fm})^3$	378(96) $\text{x}10^{-10}$	
∞	∞	684.7(6)(3.6) $\text{x}10^{-10}$	e^+e^-
∞	∞	709.0(5.1)(1.2)(2.8) $\text{x}10^{-10}$	τ decay

again statistical only.

Large finite volume effect

quark mass unphysically large

quenching error

Many imrovements under study; Long term prognosis of the method appears very promising.

Summary

$\Rightarrow$ Lattice effort is geared towards facilitating the search for signal of breakdown of Standard Model and the onset of new physics.

$\Rightarrow$ With use of domain wall quarks recently made significant progress in the longstanding issue of the $\Delta I = 1/2$ Rule.

$\Rightarrow$ Plans for the near future

- Improve our calculation of ε'/ε
- Extend use of domain wall quarks to heavy-light physics esp. for treating SU(3) breaking ratios.
- Improve lattice calculations of lowest order hadronic contribution to $(g-2)_\mu$
- Use of domain wall quarks for calculating electric dipole moment of neutron (Federico Berruto) is also being considered.

$\Rightarrow$ The anticipated boost of computing power with the arrival of QCDOC should greatly help in making progress on these issues.